JEE Advanced 2026

Sample Paper - 2 (Paper-2)

Time Allowed: 3 hours **Maximum Marks: 180**

General Instructions:

This question paper has THREE main sections and three sub-sections as below.

MCQ

- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- You will get +3 marks for the correct response and -1 for the incorrect response.

MRQ

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) the correct answer(s).
- You will get +4 marks for the correct response and -2 for the incorrect response.
- You will also get 1-3 marks for a partially correct response.

NUM

- The answer to each question is a NON-NEGATIVE INTEGER.
- You will get +4 marks for the correct response and 0 marks for the incorrect response.

Physics

1. X-rays are used in determining the molecular structure of crystalline solids because its **[3]**

a) it can penetrate the material
b) energy is high
c) its wavelength is comparable to interatomic distance
d) its frequency is low

2. Two beams of light having intensities I and 4I interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\frac{\pi}{2}$ at point A and π at point B. Then the difference between the resultant intensities at A and B is **[3]**

a) 7I
b) 4I
c) 2I
d) 5I

3. Work done in carrying a charge Q once round a circle of radius r with a charge Q at the centre is: **[3]**

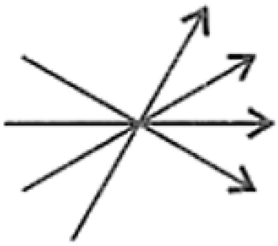
a) $\frac{QQ'}{4\pi\varepsilon_0 r}$

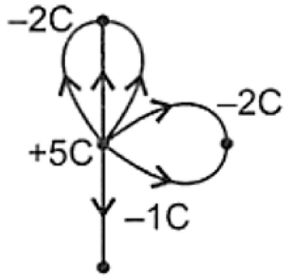
b) zero

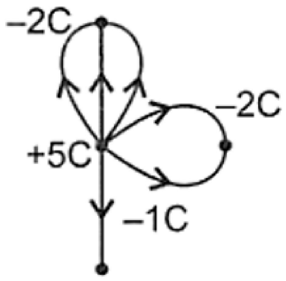
c) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

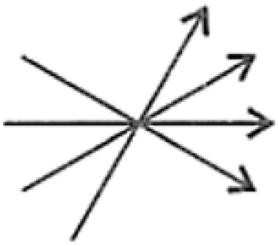
d) $\frac{QQ'}{2r}$

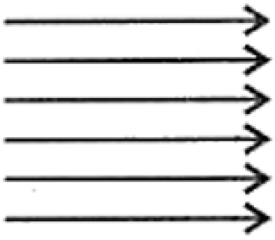
4. Which of the following configurations of electric lines of force is not possible? **[3]**

a) both  and

b) 



c) 

d) 

5. Consider a boty of mass 1.0 kg at rest at the origin at time t = 0. A force $\vec{F} = (\alpha t\hat{i} + \beta\hat{j})$ is applied on the body, where α = 1.0 Ns^{-1} and β = 1.0N. The torque acting on the body about the origin at time t = 1.0 s is $\vec{\tau}$. Which of the following statements is (are) true? **[4]**

a) The magnitude of displacement of the body at t = 1s is $\frac{1}{6}$m

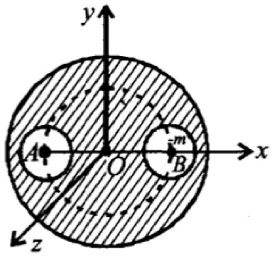
b) The torque $\vec{\tau}$ is in the direction of the unit vetor + $\hat{k}$

c) $|\vec{\tau}| = \frac{1}{3}Nm$

d) The velocity of the body at t = 1s is $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j})$ms^{-1}

6. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at A (-2, 0, 0) and B (2, 0, 0) respectively, are taken out of the solid leaving behind **[4]**

spherical cavities as shown in fig then:



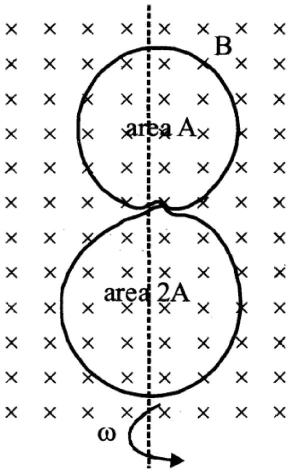
a) the gravitational force at the point B (2, 0, 0) is zero.

b) the gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$.

c) The gravitational force due to this object at the origin is zero.

d) the gravitational potential is the same at all points of circle $y^2 + z^2 = 36$.

7. A circular insulated copper wire loop is twisted to form two loops of area A and 2 A **[4]** as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At t = 0, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



a) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

b) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

c) The emf induced in the loop is proportional to the sum of the

d) The net em f induced due to both the loops is proportional areas of the two loops | to cos ωt

8. Starting at time t = 0 from the origin with speed 1 ms^{-1}, a particle follows a two-dimensional trajectory in the x-y plane so that its coordinates are related by the equation $y = \frac{x^2}{2}$. The x and y com ponents of its acceleration are denoted by a_x and a_y, respectively. Then **[4]**

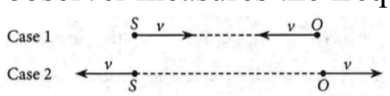
a) at t = 0, the particle's velocity points in the x-direction

b) $a_x = 0$ implies that at t = 1 s, the angle between the particle's velocity and the x axis is 45^o

c) $a_x = 1\ ms^{-2}$ implies that when the particle is at the origin, $a_y = 1\ ms^{-2}$

d) $a_x = 0$ implies $a_y = 1\ ms^{-2}$ at all times

9. A source (S) of sound has frequency 240 Hz. When the observer (O) and the source move towards each other at a speed v with respect to the ground (as shown in Case 1 in the figure), the observer measures the frequency of the sound to be 288 Hz. However, when the observer and the source move away from each other at the same speed v with respect to the ground (as shown in Case 2 in the figure), the observer measures the frequency of sound to be n Hz. The value of n is ________. **[4]**

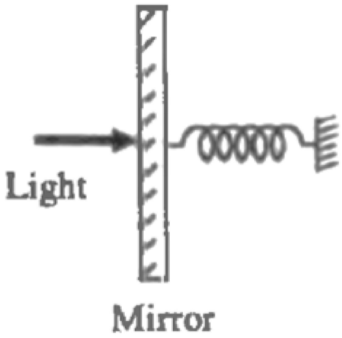


10. A ball is projected from the ground at an angle of 45^o with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30^o with the horizontal surface. The maximum height it reaches after the bounce, in metres, is ________. **[4]**

11. The dimensions of a cone are measured using a scale with a least count of 2 mm. The diameter of the base and the height are both measured to be 20.0 cm. The maximum percentage error in the determination of the volume is ________. **[4]**

12. A perfectly reflecting mirror of mass M mounted on a spring constitutes a spring-mass system of angular frequency such that $\frac{4\pi M\Omega}{h} = 10^{24}\ m^{-2}$ with h as Planck's constant. N photons of wavelength $\lambda = 8\pi \times 10^{-6}$ m strike the mirror simultaneously at normal incidence such that the mirror gets displaced by 1 μm. If the value of N is $x \times 10^{12}$, then the value of x is ________. **[4]**

[Consider the spring as massless]

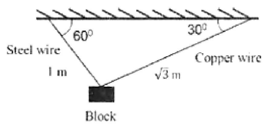


13. A horizontal pipeline carries water in a streamlined flow. At a point along the pipe, where the cross-sectional area is 10 cm^2, the water velocity is 1 ms^{-1} and the pressure is 2000 Pa. The pressure water at another point where the cross-sectional area is 5 cm^2, is ________ Pa. (Density of water = 10^3 kgm^{-3}) **[4]**

14. 300 grams of water at 25 ^{o}C is added to 100 grams of ice at 0 ^{o}C. The final temperature of the mixture is ________ ^{o}C. **[4]**

15. Consider a hydrogen-like ionised atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the n = 2 to n = 1 transition has energy 74.8 eV higher than the photon emitted in the n = 3 to n = 2 transition. The ionisation energy of the hydrogen atom is 13.6 eV. The value of Z is ________. **[4]**

16. A block of weight 100N is suspended by copper and steel wires of same cross-sectional area 0.5 cm^2 and, length $\sqrt{3}$m and 1m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with the ceiling are 30° and 60°, respectively. If elongation in copper wire is and elongation in steel wire is (Δl_S), then the ratio $\frac{\Delta l_c}{\Delta l_s}$ is ________. **[4]**
[Young's modulus for copper and steel are 1×10^{11} N/m^2 and 2×10^{11} N/m^2, respectively.]



Chemistry

17. The species having pyramidal shape is **[3]**

a) SiO_3^{2-}
b) SO_3
c) BrF_3
d) OSF_2

18. Among the following species, identify the isostructural pairs. **[3]**
$NF_3, NO_3^-, BF_3, H_3O^+, N_3H$

a) $[NF_3, H_3O^+]$ and $[N_3H, BF_3]$
b) $[NF_3, H_3O^+]$ and $[NO_3^-, BF_3]$
c) $[NF_3, NO_3^-]$ and $[BF_3, H_3O^+]$
d) $[NF_3, N_3H]$ and $[NO_3^-, BF_3]$

19. Native silver metal forms a water soluble complex with a dilute aqueous solution of NaCN in the presence of **[3]**

a) oxygen
b) nitrogen
c) carbon dioxide
d) argon

20. A molal solution is one that contains one mole of a solute in **[3]**

a) 1 L of the solution
b) 1 L of the solvent
c) 1000 g of the solvent
d) 22.4 L of the solution

21. An isotone of $^{76}_{32}Ge$ is: **[4]**

a) $^{78}_{34}Se$
b) $^{77}_{32}Ge$
c) $^{77}_{33}As$
d) $^{77}_{34}Se$

22. Choose the correct option(s) from the following **[4]**

a. Nylon-6 has amide linkages
b. Cellulose has only α-D-glucose units that are joined by glycosidic linkages
c. Teflon is prepared by heating tetrafluoroethene in presence of a persulphate catalyst at high pressure
d. Natural rubber is polyisoprene containing trans alkene units

a) Statement (a) is correct.
b) Statement (d) is correct.
c) Statement (b) is correct.
d) Statement (c) is correct.

23. For the following reaction $2X + Y \xrightarrow{k} P$ the rate of reaction is $\frac{d[P]}{dt} = k[X]$. Two moles of X are mixed with one mole of Y to make 1.0 L of solution. At 50 s, 0.5 mole of Y is left in the reaction mixture. The correct statement(s) about the reaction is(are) (Use: In 2 = 0.693) **[4]**

a) At 50 s, $-\frac{d[X]}{dt} = 13.86 \times 10^{-3}$ mol L^{-1} s^{-1}.

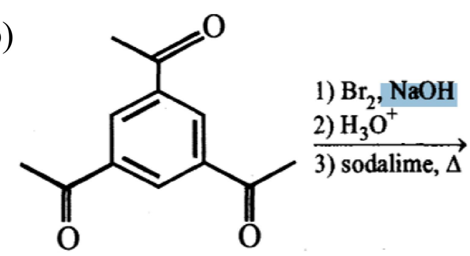
b) Half-life of X is 50s.

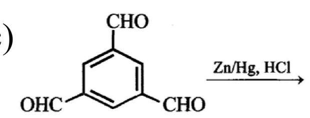
c) The rate constant, k, of the reaction is 13.86×10^{-4} s^{-1}.

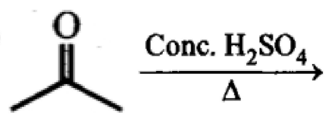
d) At 100 s, $-\frac{d[Y]}{dt} = 3.46 \times 10^{-3}$ mol L^{-1} s^{-1}.

24. The reaction(s) leading to the formation of 1, 3, 5-trimethylbenzene is (are) **[4]**

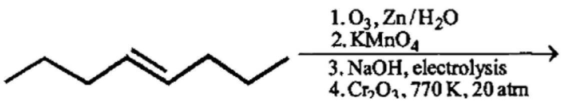
a) $Me-C\equiv C-H \xrightarrow[873\ K]{\text{heated Fe tube}}$

b) 

c) 

d) 

25. The number of -CH_2- (methylene) groups in the product formed from the following reaction sequence is ________. **[4]**



26. A trinitro compound, 1, 3, 5-tris-(4-nitrophenyl)benzene, on complete reaction with an excess of $\frac{Sn}{HCl}$ gives a major product, which on treatment with an excess of $\frac{NaNO_2}{HCl}$ at 0 °C provides P as the product. P, upon treatment with excess of H_2O at room temperature, gives the product Q. Bromination of Q in aqueous medium furnishes the product R. The compound P upon treatment with an excess of phenol under basic conditions gives the product S. The molar mass difference between compounds Q and R is 474 g mol^{-1} and between compounds P and S is 172.5 g mol^{-1}. **[4]**

The total number of carbon atoms and heteroatoms present in one molecule of S is ________.

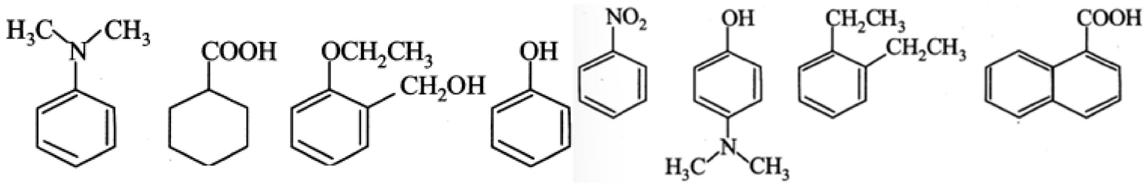
[**Use:** Molar mass (in g mol^{-1}): H = 1, C = 12, N = 14, 0 = 16, Br = 80, Cl = 35.5
Atoms other than C and H are considered as heteroatoms]

27. 20% of surface sites are occupied by N_2 molecules. The density of surface site is 6.023×10^{14} cm^{-2} and total surface area is 1000 cm^2. The catalyst is heated to 300 K while N_2 is completely desorbed into pressure of 0.001 atm and volume of 2.46 cm^3. Find the number of active sites occupied by each N_2 molecule. **[4]**

28. A decapeptide (Mol. wt. 796) on complete hydrolysis gives glycine (Mol. wt. 75), alanine and phenylalanine. Glycine contributes 47.0% to the total weight of the hydrolysed products. The number of glycine units present in the decapeptide is **[4]**

29. The normality of H_2SO_4 in the solution obtained on mixing 100 mL of 0.1 M H_2SO_4 with 50 mL of 0.1 M NaOH is ________ $\times 10^{-1}$ N. (Nearest integer) **[4]**

30. Amongst the following, the total number of compounds soluble in aqueous NaOH is **[4]**



31. The degree of dissociation is 0.4 at 400 K and 1.0 atm for the gaseous reaction $PCl_5 \rightleftharpoons PCl_3 + Cl_2$. Assuming ideal behaviour of all gases, calculate the density of equilibrium mixture at 400 K and 1.0 atmosphere. (Relative atomic mass of P = 31.0 and Cl = 35.5) **[4]**

32. What will be the resultant pH when 200mL of an aqueous solution of HCl (pH = 2.0) is mixed with 300 mL of an aqueous solution of NaOH (pH = 12.0)? **[4]**

Mathematics

33. The integer n for which $\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number, is **[3]**

a) 3 b) 2

c) 1 d) 4

34. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1, b = x^2 - 1$ and c = 2x + 1 is (are **[3]**

a) $-(2 + \sqrt{3})$ b) $4\sqrt{3}$

c) $1 + \sqrt{3}$ d) $2 + \sqrt{3}$

35. If the vertices P, Q, R of a $\triangle$PQR are rational points, which of the following points of the $\triangle$PQR is/are always rational point(s) **[3]**

a) circumcentre

b) incentre

c) centroid

d) orthocentre

36. If f (x) = sin x + cos x, g (x) = x^2 - 1, then g { f (x)} is invertible in the domain **[3]**

a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

b) $\left[0, \frac{\pi}{2}\right]$

c) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

d) $[0, \pi]$

37. Let X and Y be two arbitrary, 3 $\times$ 3, non-zero, skew-symmetric matrices and Z be an arbitrary 3 $\times$ 3, non zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric? **[4]**

a) $X^{23} + Y^{23}$

b) $X^{44} + Y^{44}$

c) $X^4Z^3 - Z^3Y^4$

d) $Y^3Z^4 - Z^4Y^3$

38. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c, respectively. Of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in at least two, and a 40% chance of passing in exactly two. Which of the following relations are true? **[4]**

a) $p + m + c = \frac{27}{20}$

b) $pmc = \frac{1}{4}$

c) $p + m + c = \frac{19}{20}$

d) $pmc = \frac{1}{10}$

39. A straight line drawn from the point $P(1,3,2)$, parallel to the line $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-6}{1}$, intersects the plane $L_1 : x - y + 3z = 6$ at the point Q. Another straight line which passes through Q and is perpendicular to the plane L_1 intersects the plane $L_2 : 2x - y + z = -4$ at the point R. Then which of the following statements is (are) TRUE? **[4]**

a) The centroid of the triangle PQR is $\left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$

b) The length of the line segment PQ is $\sqrt{6}$

c) The coordinates of R are $(1, 6, 3)$

d) The perimeter of the triangle PQR is $\sqrt{2} + \sqrt{6} + \sqrt{11}$

40. For any real numbers α and β, let $y_{\alpha,\beta}(x), x \in \mathbb{R}$, be the solution of the differential equation **[4]**

$\frac{dy}{dx} + \alpha y = xe^{\beta x}$, y(1) = 1

Let S = $\{y_{\alpha,\beta}(x) : \alpha, \beta \in \mathbb{R}\}$. Then which of the following functions belong(s) to the set S?

a) $f(x) = \frac{x^2}{2}e^{-x} + \left(e - \frac{1}{2}\right)e^{-x}$

b) $f(x) = \frac{e^x}{2}\left(x - \frac{1}{2}\right) + \left(e - \frac{e^2}{2}\right)e^{-x}$

c) $f(x) = \frac{e^x}{2}\left(\frac{1}{2} - x\right) + \left(e + \frac{e^2}{2}\right)e^{-x}$

d) $f(x) = -\frac{x^2}{2}e^{-x} + \left(e + \frac{1}{2}\right)e^{-x}$

41. Let f : [1, ∞) → [2, ∞) be a differentiable function such that f(1) = 2. If $6\int_1^x f(t)dt$ = 3xf(x) - x^3 for all x ≥ 1, then the value of f(2) is **[4]**

42. The smallest value of k, for which both the roots of the equation x^2 - 8kx + 16 (k^2 - k + 1) = 0 are real, distinct, and have values at least 4, is **[4]**

43. Let $\omega = e^{\frac{i\pi}{3}}$ and a, b, c, x, y, z be non-zero complex numbers such that a + b + c = x, $a + b\omega + c\omega^2 = y$, $a + b\omega^2 + c\omega$ = z. Then, the value of $\frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}$ is **[4]**

44. The value of $\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$ in the interval $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$ equals ________. **[4]**

45. For x ∈ ℝ, then number of real roots of the equation $3x^2 - 4\left|x^2 - 1\right| + x - 1 = 0$ is ________. **[4]**

46. Let $l_1, l_2, ..., l_{100}$ be consecutive terms of an arithmetic progression with common difference d_1 and let $w_1, w_2 .., w_{100}$ be consecutive terms of another arithmetic progression with common difference d_2, where d_1, d_2 = 10. For each i = 1, 2,..., 100, let R_i be a rectangle with length I_i width w_i and area A_i. If $A_{51} - A_{50}$ = 1000, then the value of $A_{100} - A_{90}$ is ________. **[4]**

47. A group of 9 students, $s_1, s_2, \ldots\ldots, s_9$, is to be divided to form three teams X, Y, and Z of sizes 2, 3, and 4, respectively. Suppose that s_1 cannot be selected for the team X, and s_2 cannot be selected for the team Y. Then the number of ways to form such teams, is ________. **[4]**

48. The number of values of θ in the interval, $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0, ±1, ±2 and tan θ = cot 5θ as well as sin 2θ = cos 4θ is **[4]**

Solution

Physics

1.

(c) its wavelength is comparable to interatomic distance

Explanation:

The crystal structure is explored through the diffraction of waves having a wavelength comparable with the interatomic spacing (10^{-10} m) in crystals. Radiation of longer wavelength cannot resolve the details of structure, while radiation of much shorter wavelength is diffracted through inconveniently small angles. Usually, diffraction of X-ray is employed in the study of crystal structure as X-rays have a wavelength comparable to interatomic spacing.

2.

(b) 4I

Explanation:

As we know, $I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

When phase difference is $\frac{\pi}{2}$

$I_{\pi/2} = I + 4I \Rightarrow I_{\pi/2} = 5I$

Again when d phase difference is π

$I_{\pi} = I + 4I + 2\sqrt{I}\sqrt{4I}\cos\pi = I$

$\therefore I_{\pi/2} - I_{\pi} = 5I - I = 4I$

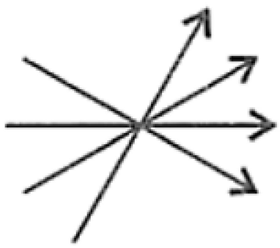
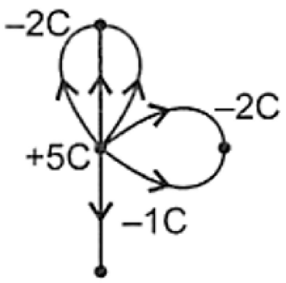
3.

(b) zero

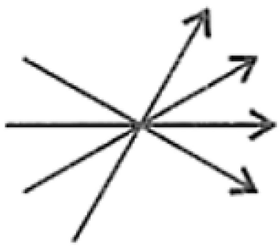
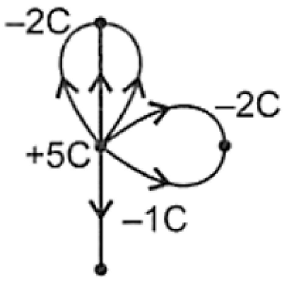
Explanation:

Work done on equilibrium line/surface is zero.

4. **(a)**

both  and 

Explanation:

both  and 

5. **(c)** $|\vec{\tau}| = \frac{1}{3}Nm$

(d) The velocity of the body at t = 1s is $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j})\text{ms}^{-1}$

Explanation: Given $\vec{F} = \alpha t\hat{i} + \beta\hat{j}$ or $\vec{F} = t\hat{i} + \hat{j}$ ($\because \alpha = 1NS^{-1}$ and $\beta = 1N$))

$\therefore \frac{md\vec{v}}{dt} = t\hat{i} + \hat{j}$

$\therefore d\vec{v} = tdt\hat{i} + dt\hat{j}$ $[\because m = 1]$

$\therefore \int_0^v d\vec{v} = \int_0^t tdt\hat{i} + \int_0^t dt\hat{j}$

$\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$

At t = 1s, $\vec{v} = \frac{1}{2}\hat{i} + \hat{j} = \frac{1}{2}(\hat{i} + 2\hat{j})\text{ms}^{-1}$

Also, $\vec{v} = \frac{d\vec{r}}{dt} = \frac{t^2}{2}\hat{i} + t\hat{j}$

$\therefore d\vec{r} = \frac{t^2}{2}dt\hat{i} + tdt\hat{j}$

or, $\int_0^r d\vec{r} = \int_0^t \frac{t^2}{2}dt\hat{i} + \int_0^t tdt\hat{j}$

$\therefore \vec{r} = \frac{t^3}{6}\hat{i} + \frac{t^2}{2}\hat{j}$

At t = 1, $\vec{r} = \frac{1}{6}\hat{i} + \frac{1}{2}\hat{j} \therefore |\vec{r}| = \sqrt{\frac{1}{36} + \frac{1}{4}} = \sqrt{\frac{10}{36}}$

$\vec{\tau} = \vec{r} \times \vec{F} = \left(\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right) \times (\hat{i} + \hat{j})$ (at t = 1 s)

or, $\vec{\tau} = -\frac{1}{3}\hat{k} \therefore |\vec{\tau}| = \frac{1}{3}$Nm

6. **(b)** the gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$.

(c) The gravitational force due to this object at the origin is zero.

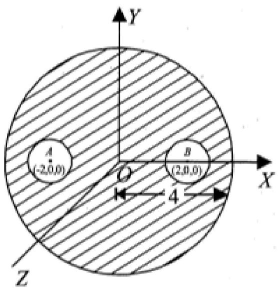
(d) the gravitational potential is the same at all points of circle $y^2 + z^2 = 36$.

Explanation: The gravitational field (E) intensity at the point O i.e., centre of a solid sphere is zero. Force acting on a test mass m0 placed at O

$F = m_0E = m_0 \times 0 = 0$

The gravitational field due to masses at A and B at ‘O’ is equal and opposite.

Now, $y^2 + z^2 = 36$ represents the equation of a circle with centre (0, 0, 0) and radius 6 units the plane of the circle is perpendicular to x-axis. As the plane of these circles is Y-Z $\perp$ to X-axis so potential at any point on these two circles will be constant due to mass M and masses at A and B.



7. **(a)** The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

(b) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

Explanation: The net magnetic flux through the loops at time t

ϕ = B.2 A cos ωt - BA cos ωt = B(2A - A)cos ωt = BA cos ωt

$\therefore \left|\frac{d\phi}{dt}\right| = B\omega A\ \sin\omega t$

So, $\left|\frac{d\phi}{dt}\right|$ is maximum, when $\phi = \omega t = \frac{\pi}{2}$

The emf induced in the smaller loop,

$\varepsilon_{\text{smaller}} = -\frac{d}{dt}(BA\cos\omega t) = B\omega A\ \sin\omega t$

$\therefore$ Amplitude of maximum net emf induced in both the loops

= Amplitude of maximum emf induced in the smaller loop alone.

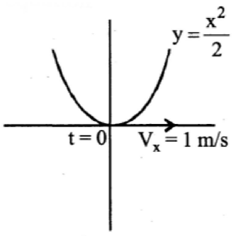
8. **(a)** at t = 0, the particle's velocity points in the x-direction

(b) $a_x = 0$ implies that at t = 1 s, the angle between the particle's velocity and the x axis is 45^o

(c) $a_x = 1\ ms^{-2}$ implies that when the particle is at the origin, $a_y = 1\ ms^{-2}$

(d) $a_x = 0$ implies $a_y = 1\ ms^{-2}$ at all times

Explanation: According to question, eauation



$y = \frac{x^2}{2}$

At t = 0, $\left.\begin{array}{l} x = 0, y = 0 \\ u = 1 \end{array}\right\}$ given

$y = \frac{x^2}{2}$

$\frac{dy}{dt} = \frac{1}{2} \cdot 2x\ \frac{dx}{dt} = x\ \frac{dx}{dt} \Rightarrow v_y = xv_x$

Now differenciate wrt time

$\frac{d^2y}{dt^2} = x\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2$

$a_y = \frac{dx}{dt} \cdot V_x + xa_x$

$a_y = v_x^2 + xa_x$

- If $a_x = 1$ and particle is at origin $(x = 0, y = 0)$

  $a_y = v_x^2$

  $a_y = 1^2 = 1 \text{ m/s}^2$
- $a_x = 0$

  $a_y = v_x^2 + xa_x \Rightarrow a_y = v_x^2$

  If $a_x = 0$, v_x = constant $= 1 \Rightarrow a_y = 1^2 = 1$
- At $t = 0$, $x = 0$ $v_y = xv_x$

  Speed = 1; $v_y = 0 \Rightarrow v_x = 1$
- $a_x = 0$ implies that at $t = 1$s

  $a_y = v_x^2 + xa_x \Rightarrow v_y = xv_x \Rightarrow a_y = v_x^2$

  If $a_x = 0 \Rightarrow V_x$ = constant initially $(v_x = 1) \Rightarrow a_y = 1^2 = 1$

  At t = 1 sec

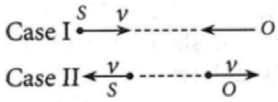
  $v_y = 0 + a_y \times t = 1 \times 1 = 1$

  $\tan\theta = \frac{v_y}{v_x} = x$ ($\theta \to$ angle with x axis)

  $\therefore \tan\theta = \frac{v_y}{v_x} = \frac{1}{1} = 1 \Rightarrow \theta = 45^o$

9. 200

   Explanation:

   For case I:

   $f' = \left(\frac{c+v}{c-v}\right) f$

   $288 = 240\left(\frac{c+v}{c-v}\right)$ ...(i)

   For case II: $f' = f\left(\frac{c-v}{c+v}\right) \Rightarrow n = 240\left(\frac{c-v}{c+v}\right)$ ...(ii)

   From (i) & (ii)

   $288 \times n = 240 \times 240\left(\frac{c+v}{c-v}\right)\left(\frac{c-v}{c+v}\right)$

   $288n = 240 \times 240 \Rightarrow n = 200\ Hz$

10. 30

    Explanation:

    Maximum height,

    $H = \frac{u^2 \sin^2\theta}{2g} \Rightarrow 120 = \frac{u^2\left(\frac{1}{2}\right)}{2g}$

    $\therefore u^2 = 480$ g

    Upon hitting the ground, it loses half of its kinetic energy

    $\therefore K.E_{initial} = \frac{1}{2} mu^2 = 240$ mg

    $K.E_{final} = \frac{1}{2}(240 \text{ mg}) = 120$ mg

$\therefore \frac{1}{2}mv^2 = 120\ mg$

$\therefore v^2 = 240\ g$

After the bounce, the maximum height the ball reaches

$\therefore H' = \frac{v^2 \sin^2\theta}{2g} = \frac{240g \times \left(\frac{1}{4}\right)}{2g} = 30$ m

11. 3

Explanation:

$V = \frac{1}{3}\pi r^2 h; V = \frac{1}{3}\pi \times \frac{D^2}{4} \cdot h$

$V = \frac{1}{12}\pi D^2 h; \frac{dV}{V} \times 100 = \left(\frac{2.d(D)}{D} + \frac{dh}{h}\right) \times 100$

$= \left(2 \times \frac{0.2}{20} + \frac{0.2}{20}\right) \times 100 = 3\%$

12. 1

Explanation:

From conservation of momentum principle, change in momentum of photon = change in momentum of mirror

$2(NP) = MV_{max}$

$\Rightarrow 2\left[N\left(\frac{h}{\lambda}\right)\right] = MV_{max}$

$\therefore 2\frac{Nh}{\lambda} = M(A\Omega)$ ... $[V_{max} = A\Omega]$

$N = \left(\frac{M\Omega}{h}\right)\frac{A\lambda}{2} = \frac{10^{24}}{4\pi} \times \frac{10^{-6} \times 8\pi \times 10^{-6}}{2}$

$\left[\because \frac{m\Omega}{h} = \frac{10^{24}}{4\pi}; A = 1hm; \lambda = 8\pi \times 10^{-6}\right]$

$\therefore N = 1 \times 10^{12} = x \times 10^{12}$

$\therefore x = 1$

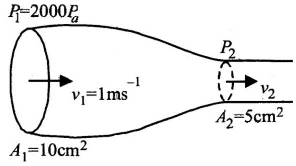
13. 500

Explanation:

According to equation of continuity

$A_1v_1 = A_2v_2 \Rightarrow 10 \times 1 = 5 \times v_2 \Rightarrow v_2 = 2$m/s

Now, using Bernoulli's theorem



$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$\Rightarrow 2000 + \frac{1}{2} \times 1000 \times 1^2$

$= P_2 + \frac{1}{2} \times 1000 \times 2^2$

$\therefore P_2 = 500$ Pa

14. 0

Explanation:

The heat required for 100 g of ice at 0^{O} C to change into water at 0^{O} C = mL = 100 × 80 × 4.2 = 33,600 J

The heat released by 300 g of water at 25^{O} C to change its temperature to 0^{O} C = mcΔT = 300 × 4.2 × 25 = 31,500 J

Hence complete ice will not melt, so the final temperature of the mixture will be 0^{O}C.

15. 3

Explanation:

$\Delta E_{2-1} = 13.6 \times Z^2 \left[1 - \frac{1}{4}\right] = 13.6 \times Z^2 \left[\frac{3}{4}\right]$

$\Delta E_{3-2} = 13.6 \times Z^2 \left[\frac{1}{4} - \frac{1}{9}\right] = 13.6 \times Z^2 \left[\frac{5}{36}\right]$

$\therefore \Delta E_2 = \Delta E_{3-2} + 74.8$

$13.6 \times Z^2 \left[\frac{3}{4}\right] = 13.6 \times Z^2 \left[\frac{5}{36}\right] + 74.8$

$13.6 \times Z^2 \left[\frac{3}{4} - \frac{5}{36}\right] = 74.8$

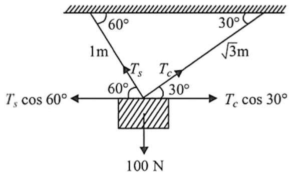
$Z^2 = 9$

$\therefore Z = 3$

16. 2

Explanation:

Given: $l_C = \sqrt{3}$ m; $l_S = 1$ m; $Y_C = 1 \times 10^{11}$ N/m^2 and T = 2 × 10^{11} N/m^2.



At equilibrium, $T_S \cos 60° = T_C \cos 30°$

$\Rightarrow \frac{T_S}{2} = \frac{T_c\sqrt{3}}{2} \Rightarrow T_3 = \sqrt{3}T_r \Rightarrow \frac{T_c}{T_s} = \frac{1}{\sqrt{3}}$

$\therefore \frac{l_c}{l_s} = \frac{\sqrt{3}}{1}$ and $\frac{Y_c}{Y_s} = \frac{1\times10^{11}}{2\times10^{11}} = \frac{1}{2}$

From, $Y = \frac{Fl}{A\Delta l} \Rightarrow \Delta l = \frac{Fl}{AY}$

Here, $A_S = A_C$

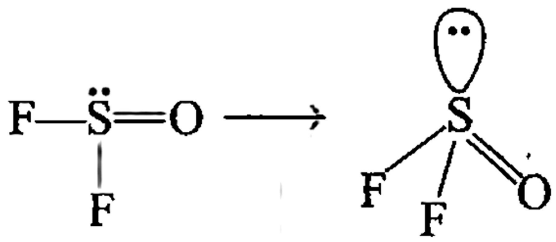
$\therefore \frac{\Delta l_c}{\Delta l_s} = \left(\frac{T_c}{T_s}\right) \times \left(\frac{l_c}{l_s}\right) \times \left(\frac{Y_s}{Y_c}\right) = \left(\frac{1}{\sqrt{3}}\right) \times \left(\frac{\sqrt{3}}{1}\right) \times \left(\frac{2}{1}\right) = 2$

Chemistry

17.

(d) OSF_2

Explanation:



S is sp^3-hybridised **Pyramidal**

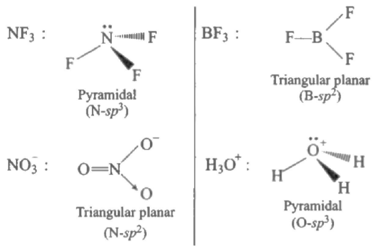
SO_3 is planar (S is sp^2-hybridised), BrF_3 is T-shaped and SiO_3^{2-} is planar (Si is sp^2-

hybridised).

18.

(b) $[NF_3, H_3O^+]$ and $[NO_3^-, BF_3]$

Explanation:



$N_3H: \ :\bar{\ddot{N}} = \overset{+}{N} = \ddot{N} - H$

Central nitrogen is *sp*-hybridised

Therefore, NF_3,H_3O^+ and BF_3,NO_3^- pairs have same shape.

19. **(a)** oxygen

Explanation:

In the presence of oxygen, Ag metal forms a water soluble complex Na $[Ag(CN)_2]$ with dilute solution of NaCN

$$4Ag + 8NaCN + 2H_2O + O_2 \longrightarrow \underset{(Soluble)}{4Na[Ag(CN)_2]} + 4NaOH$$

20.

(c) 1000 g of the solvent

Explanation:

1000 g of the solvent

21. **(a)** $^{78}_{34}Se$

(c) $^{77}_{33}As$

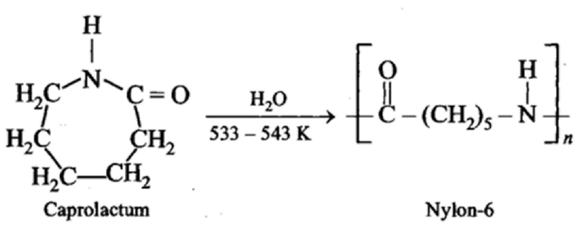
Explanation: $^{77}_{33}As$ and $^{78}_{34}Se$ have same number of neutrons (= A - Z) as $^{76}_{32}Ge$.

22. **(a)** Statement (a) is correct.

(d) Statement (c) is correct.

Explanation:

a. Nylon-6. It is obtained by heating caprolactam with water at high temperature and have amide linkage.



b. Cellulose has only β-D-glucose units that are joined by glycosidic linkages between C - 1 of one glucose unit and C - 4 of the next glucose unit.

c. Teflon is prepared by heating tetrafluoroethene in presence of a persulphate catalyst at high pressure.

$$\underset{\text{Tetrafluoroethene}}{CF_2 = CF_2} \xrightarrow[\text{High pressure}]{\text{Catalyst}} \underset{\text{Teflon}}{-[CF_2 - CF_2]-}$$

d. Natural rubber is a linear polymer of isoprene (2-methyl-1, 3-butadiene) containing cis alkene units. It is also called cis-1, 4-polyisoprene.

23. **(a)** At 50 s, $-\frac{d[X]}{dt} = 13.86 \times 10^{-3}$ mol L^{-1} s^{-1}.

(b) Half-life of X is 50s.

(d) At 100 s, $-\frac{d[Y]}{dt} = 3.46 \times 10^{-3}$ mol L^{-1} s^{-1}.

Explanation: Rate $= \frac{dP}{dt} = k[X]^1$

$$\begin{array}{llll} & 2X & + \ Y & \rightarrow \ P \\ t=0 & & 2\ mol & 1\ mol \\ t=50\ s & & 1\ mol & 0.5\ mol \end{array}$$

$$-\frac{1}{2}\frac{d[X]}{dt} = \frac{d[P]}{dt} = k[X]^1$$

$$-\frac{d[X]}{dt} = 2k[X]^1$$

$$2k = \frac{\ln 2}{50} \Rightarrow k = \frac{\ln 2}{100} = 6.93 \times 10^{-3}\ s^{-1}$$

$$t_{1/2} = \frac{\ln 2}{2k} = \frac{\ln 2 \times 50}{\ln 2} = 50 \text{ sec}$$

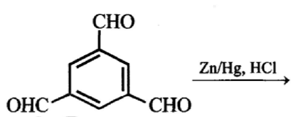
$$\text{At 50 sec } \frac{-d[X]}{dt} = 2k \times (1)^1 = \frac{\ln 2}{50} = 13.86\ 10^{-3} \text{mol } L^{-1}\ s^{-1}$$

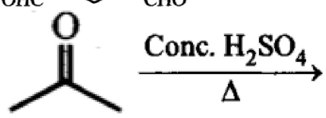
$$\text{At 100 sec } -\frac{1}{2}\frac{d[X]}{dt} = \frac{-d[Y]}{dt}$$

$$\Rightarrow -\frac{d[Y]}{dt} = \frac{\ln 2}{100} \times \frac{1}{2}\left\{\frac{-d[Y]}{dt} = k[X]^1\right\}$$

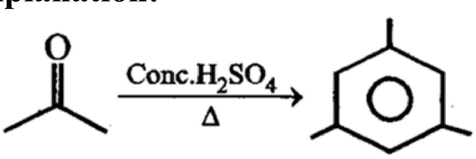
$$\frac{d[Y]}{dt} = 3.46 \times 10^{-3} \text{ mol } L^{-1}\ s^{-1}$$

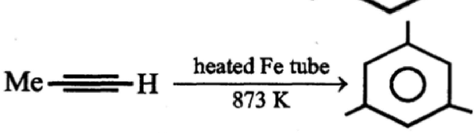
24. **(a)** $Me-C \equiv C-H \xrightarrow[873\ K]{\text{heated Fe tube}}$

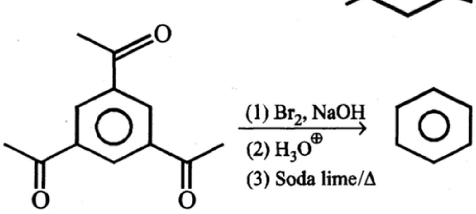
(c) 

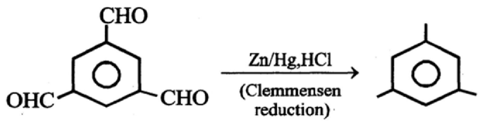
(d) 

Explanation:

a. 

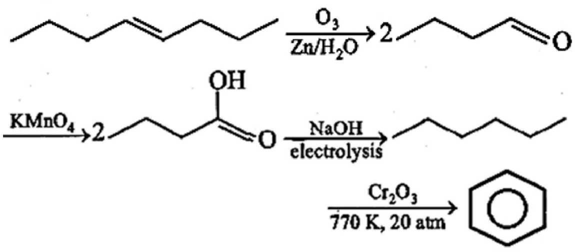
b. 

c. 

d. 

25. 0.0

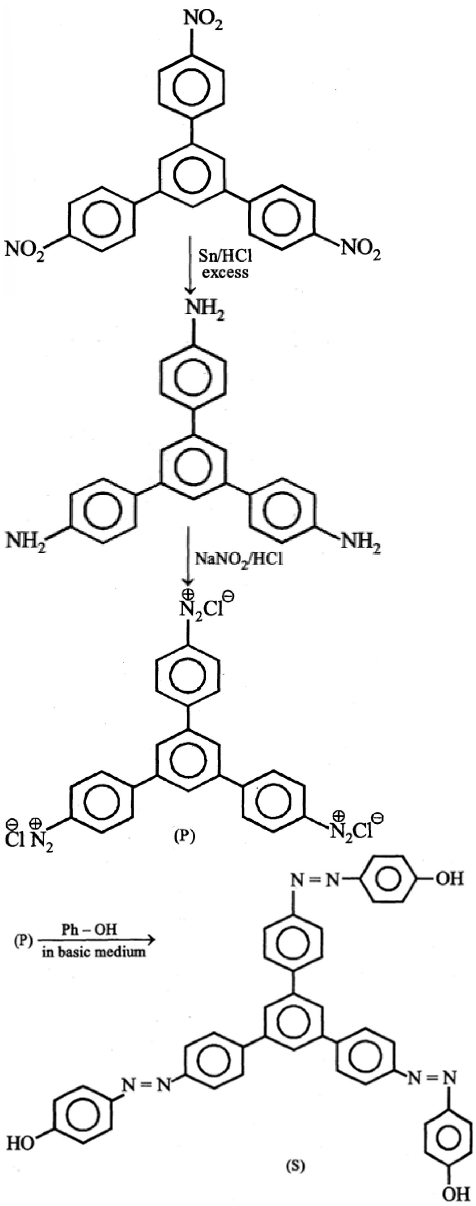
Explanation:



Number of -CH_2- groups in the product = 0.

26. 51.0

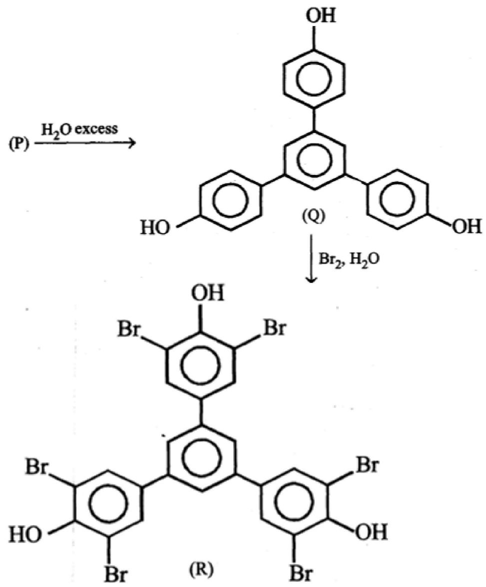
Explanation:



Number of carbon atoms = 42

Number of hetero atoms = 09

Total = 51



Number of hetero atoms in R is 9.

27. 2

Explanation:

P_{N_3} = 0.001 at, T = 300 K, V = 2.46 cm^2

∴ Number of N_2 molecules

$= \frac{PV}{RT} \times N_A = \frac{0.001 \times 2.46 \times 10^{-3}}{0.0821 \times 300} \times 6.023 \times 10^{23}$

$= 6.016 \times 10^{16}$

Now, the total number of surface sites = Density × Total surface area

$= 6.023 \times 10^{14} \times 1000 = 6.023 \times 10^{17}$

Sites occupied by N_2 molecules $= \frac{20}{100} \times 6.023 \times 10^{17}$

$= 12.04 \times 10^{16}$

∴ No. of sites occupied by each N_2 molecule

$= \frac{12.04 \times 10^{16}}{6.016 \times 10^{16}} \approx 2$

28. 6

Explanation:

Molecular weight of decapeptide = 796 g/mol

Total bonds to be hydrolysed = (10 - 1) = 9 per molecule

Total weight of H_2O added = 9 × 18 = 162 g/mol

Total weight of hydrolysis products = 796 + 162 = 958 g

Total weight % of glycine (given) = 47%

Total weight of glycine in product $= \frac{958 \times 47}{100}$ g = 450 g

Molecular weight of glycine = 75 g/mol

Number of glycine molecules $= \frac{450}{75} = 6$

NOTE: A dipeptide has one peptide bond. Thus, a decapeptide has 9 peptide bonds.

29. 1.0

Explanation:

No. of equivalents of H_2SO_4 = 100 × 0.1 × 2 = 20

No. of equivalents of NaOH = 50 × 0.1 = 5

No. of equivalents of H_2SO_4 left = 20 - 5 = 15

Total volume = (100 + 50) = 150 mL ⇒ 150 × x = 15

$x = \frac{1}{10} = 0.1N = 1 \times 10^{-1}$ N

30. 4.0

Explanation:

All carboxylic acids and phenols are soluble in aqueous NaOH. Thus, four compounds are soluble in aqueous NaOH.

31. 4.53

Explanation:

	PCl_5	⇌	PCl_3	+	Cl_2
Initial moles	1		0		0
Moles at eq.	1–0.4		0.4		0.4

∴ Total moles at equilibrium = 1 - 0.4 + 0.4 + 0.4 = 1.4

Also $\frac{\text{Normal mol. wt. of } PCl_5}{\text{Exp.mol. wt. of } PCl_5} = 1 + \alpha = 1.4$

or $\frac{208.5}{\text{Exp. mol. wt. of } PCl_5} = 1.4$

∴ Exp. mol. wt. of PCl_5 or m. wt. of mixture = $\frac{208.5}{1.4}$

Now using, PV = $\frac{w}{m}$RT for mixture

$d = \frac{w}{V} = \frac{Pm}{RT} = \frac{1 \times 208.5}{1.4 \times 0.082 \times 400} = 4.53$ g/litre

32. 11.30

Explanation:

	HCl	+ NaOH	⟶ NaCl	+ H_2O
Meq. before reaction	$200 \times 10^{2-}$	$300 \times 10^{2-}$		
Meq. after reaction	0	$100 \times 10^{2-}$	$200 \times 10^{2-}$	$200 \times 10^{2-}$

pH of HCl = 2, pH of NaOH = 12

∴ [HCl] = 10^{-2} M, ∴ [NaOH] = 10^{-2} M

∴ $[OH^-] = \frac{100 \times 10^{-2}}{500} = 2 \times 10^{-3}$ or P[OH] = - log(2 × 10^{-1})

∴ pOH = 2.6989; ∴ pH = 11.3010

[pH = 14 - p(OH)]

Mathematics

33. **(a)** 3

Explanation:

$$\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$$

$$= \lim_{x \to 0} \frac{\left(-2\sin^2\frac{x}{2}\right)\left\{\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\right) - \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\right)\right\}}{x^n}$$

$= \lim_{x \to 0} \frac{\left(-2\sin^2\frac{x}{2}\right)\left(-x-\frac{2x^2}{2!}-\frac{x^3}{3!}-\ldots\right)}{4\left(\frac{x}{2}\right)^2 x^{n-2}}$

$= \lim_{x \to 0} \frac{\sin^2\frac{x}{2}\left(1+x+\frac{x^2}{3!}+\ldots\right)}{2\left(\frac{x}{2}\right)^2 x^{n-3}}$

Above limit is finite, if n - 3 = 0, i.e. n = 3.

34.

(c) $1+\sqrt{3}$

Explanation:

Given: $a = x^2+x+1, b = x^2-1$, c=2 x+1

and $\angle C = \frac{\pi}{6}$

Now, $\cos C = \frac{a^2+b^2-c^2}{2ab}$ [cosine nule]

$\Rightarrow \cos\frac{\pi}{6} = \frac{(x^2+x+1)^2+(x^2-1)^2-(2x+1)^2}{2(x^2+x+1)(x^2-1)}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{(x^2+3x+2)(x^2-x)+(x^2-1)^2}{2(x^2+x+1)(x^2-1)}$

$\Rightarrow \sqrt{3}\left(x^2+x+1\right)(x+1)(x-1) = x(x-1)(x+1)(x+2)+(x+1)^2(x-1)^2$

$\Rightarrow (x+1)(x-1)\left[\sqrt{3}\left(x^2+x+1\right)-x(x+2)-(x+1)(x-1)\right] = 0$

$\Rightarrow (x+1)(x-1)\left[(\sqrt{3}-2)x^2+(\sqrt{3}-2)x+(\sqrt{3}+1)\right] = 0$

$\therefore x = -1, 1, (\sqrt{3}+1), -(\sqrt{3}+2)$

Now for $x = -1$ and $1, b = 0$ which is not possible

and for $x = -(2+\sqrt{3}), c = -4-2\sqrt{3}+1 < 0$, which is not possible.

Hence, $x = \sqrt{3}+1$

35.

(c) centroid

Explanation:

Since, the coordinates of the centroid are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$, then the centroid is always a rational point.

36.

(c) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

Explanation:

By definition of composition of function,

g (f (x)) = (sin x + cos x)^2 - 1, is invertible

(i.e. bijective)

$\Rightarrow$ g {f (x)} = sin 2x is bijective.

We know, sin x is bijective, only when x $\in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Thus, g$\{$f(x)$\}$ is bijective, if $-\frac{\pi}{2} \leq 2x \leq \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

37. **(a)** $X^{23} + Y^{23}$

(c) $X^4Z^3 - Z^3Y^4$

Explanation: X' = -X, Y' = -Y, Z' = Z

$(Y^3Z^4 - Z^4Y^3)' = (Z^4)'(Y^3)' - (Y^3)'(Z^4)'$

$= (Z')^4(Y')^3 - (Y')^3(Z')^4$

$= -Z^4Y^3 + Y^3Z^4 = Y^3Z^4 - Z^4Y^3$

Therefore $(Y^3Z^4 - Z^4Y^3)$ is a symmetric matrix.

Similarly $X^{44} + Y^{44}$ is a symmetric matrix and $X^4Z^3 - Z^3X^4$ and $X^{23} + Y^{23}$ are skew symmetric matrices.

38. **(a)** $p + m + c = \frac{27}{20}$

(d) $pmc = \frac{1}{10}$

Explanation: P(Passing atleast in one subject)

$= P(P \cup C \cup M) = 1 - P(\overline{P \cup C \cup M}) = 0.75$

$\Rightarrow P \cdot (\bar{P}) \cdot P(\bar{C}) \cdot P(\bar{M}) = 1 - 0.75 = 0.25 = \frac{1}{4}$

$\Rightarrow$ (1 - m)(1 - P)(1 - C) = $\frac{1}{4}$ ...(i)

P(Passing exactly in two subjects) = 0.4

$\Rightarrow P(P \cap C \cap \bar{M}) + P(P \cap \bar{C} \cap M) + P(\bar{P} \cap C \cap M) = \frac{2}{5}$

$\Rightarrow$ P·C(1 - m) + pm(1 - c) + cm(1 - p) = $\frac{2}{5}$ ...(ii)

P(Passing atleast in two subject) = 0.5

$\Rightarrow$ Pm(1 - c) + Pc(1 - m) + cm(1 - P) + Pcm = $\frac{1}{2}$ ...(iii)

$\Rightarrow pcm = \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$ [From (ii)]

$\therefore pmc = \frac{1}{10}$ is true.

from (i), (ii) and (iii), we get

p + c + m = $\frac{27}{20}$

$\therefore p + m + c = \frac{27}{20}$ is true.

39. **(a)** The centroid of the triangle PQR is $\left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$

(b) The length of the line segment PQ is $\sqrt{6}$

Explanation: The line is $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z-2}{1} = t$

$(x, y, z) \equiv (t + 1, 2t + 3, t + 2)$

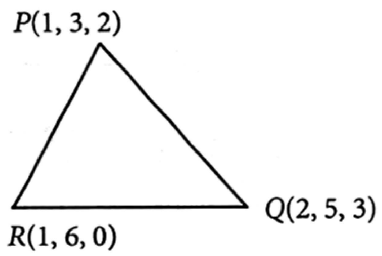
It meets $x - y + 3z = 6$ in $(t + 1) - (2t + 3) + 3(t + 2) = 6$, giving $t = 1$

$\therefore Q = (2, 5, 3)$

Line $\frac{x-2}{1} = \frac{y-5}{-1} = \frac{z-3}{3} = u$ meets $2x - y + z = -4$ in $(u + 2, -u + 5, 3u + 3)$ giving

$2(u + 2) - (5 - u) + (3u + 3) = -4$. So, $u = -1$

Hence, R is $(1, 6, 0)$



Now, $PQ = \sqrt{6}$

$QR = \sqrt{11}$

$RP = \sqrt{13}$

Hence, the perimeter is

$\sqrt{6} + \sqrt{11} + \sqrt{13}$

Centroid is $\left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$

40. **(a)** $f(x) = \frac{x^2}{2}e^{-x} + \left(e - \frac{1}{2}\right)e^{-x}$

(b) $f(x) = \frac{e^x}{2}\left(x - \frac{1}{2}\right) + \left(e - \frac{e^2}{2}\right)e^{-x}$

Explanation: Integrating factor $= e^{\int \alpha dx} = e^{\alpha x}$

Solution: $ye^{\alpha x} = \int xe^{(\alpha+\beta)x}dx$

Case I:

If $\alpha + \beta = 0$ then $ye^{\alpha x} = \frac{x^2}{2} + C$

Put x = 1 and y = 1

$\Rightarrow$ C $= e^{\alpha} - \frac{1}{2}$

So, $ye^{\alpha x} = \frac{x^2}{2} + e^{\alpha} - \frac{1}{2}$

$\Rightarrow y = \frac{x^2}{2} \cdot e^{-\alpha x} + \left(e^{\alpha} - \frac{1}{2}\right)e^{-\alpha x}$

for $\alpha = 1$

$y = \frac{x^2}{2}e^{-x} + \left(e - \frac{1}{2}\right)e^{-x}$

$f(x) = \frac{x^2}{2}e^{-x} + \left(e - \frac{1}{2}\right)e^{-x}$ is correct.

Case II:

If $\alpha + \beta \neq 0$

$ye^{\alpha x} = \frac{x \cdot e^{(\alpha+\beta)x}}{\alpha+\beta} - \frac{1}{\alpha+\beta}\int e^{(\alpha+\beta)x}dx$

$\Rightarrow ye^{\alpha x} = \frac{xe^{(\alpha+\beta)x}}{\alpha+\beta} - \frac{e^{(\alpha+\beta)x}}{(\alpha+\beta)^2} + C$

Put x = 1 and y = 1, we get

$c = e^{\alpha} - \frac{e^{\alpha+\beta}}{\alpha+\beta} + \frac{e^{\alpha+\beta}}{(\alpha+\beta)^2}$

y $= \frac{e^{\beta x}}{(\alpha+\beta)^2}((\alpha+\beta)x - 1) + e^{-\alpha x}\left(e^{x} - \frac{e^{\alpha+\beta}}{\alpha+\beta} + \frac{e^{\alpha+\beta}}{(\alpha+\beta)^2}\right)$

For $\alpha = \beta = 1$

$\frac{e^x}{4}(2x - 1) + e^{-x}\left(e - \frac{e^2}{2} + \frac{e^2}{4}\right)$

$y = \frac{e^x}{4}\left(x - \frac{1}{2}\right) + c^{-x}\left(c - \frac{e^2}{4}\right)$

So, $f(x) = \frac{e^x}{2}\left(x - \frac{1}{2}\right) + \left(e - \frac{e^2}{2}\right)e^{-x}$ is correct.

41. 6

Explanation:

$6\int_1^x f(t)dt = 3xf(x) - x^3$

On differentiating, we get $6f(x) = 3f(x) + 3xf'(x) - 3x^2$

$\Rightarrow f'(x) - \frac{1}{x}f(x) = x$, I.F. $= \frac{1}{x}$

$\therefore$ Solution is $f(x)\frac{1}{x} = \int 1.dx = x + c$

$\therefore f(x) = x^2 + cx$

But $f(1) = 2 \Rightarrow c = 1$, $\therefore f(x) = x^2 + x$

$\Rightarrow f(2) = 4 + 2 = 6$

42. 2

Explanation:

i. Given, $x^2 - 8kx + 16(k^2 - k + 1) = 0$

Now, $D = 64\left\{k^2 - \left(k^2 - k + 1\right)\right\} = 64(k - 1) > 0$

$k > 1$

ii. $-\frac{b}{2a} > 4 \Rightarrow \frac{8k}{2} > 4$

$\Rightarrow k>1$

iii. $f(4) \geq 0$

$\Rightarrow 16 - 32k + 16(k^2 - k + 1) > 0$

$\Rightarrow k^2 - 3k + 2 \geq 0$

$\Rightarrow (k - 2)(k - 1) \geq 0$

$\Rightarrow k \leq 1$ or $k \geq 2$

Hence, $k = 2$

43. 3

Explanation:

$\frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2} = \frac{x\bar{x}+y\bar{y}+z\bar{z}}{|a|^2+|b|^2+|c|^2}$

$(a + b + c)\left(\bar{a} + \bar{b} + \bar{c}\right) + (a + b\omega + c\omega^2)$

$= \frac{\left(\bar{a}+\bar{b}\omega^2+\bar{c}\omega\right)+\left(a+b\omega^2+c\omega\right)\left(\bar{a}+\bar{b}\omega+\bar{c}\omega^2\right)}{|a|^2+|b|^2+|c|^2}$

$= \frac{3\left(|a|^2+|b|^2+|c|^2\right)}{|a|^2+|b|^2+|c|^2} = 3$

44. 0

Explanation:

$\sec^{-1}\left[\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right]$

$= \sec^{-1}\left[\frac{1}{2}\sum_{k=0}^{10}\frac{1}{2\cos\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\cos\left(\frac{7\pi}{12}+\frac{k\pi}{2}+\frac{\pi}{2}\right)}\right]$

$= \sec^{-1}\left[\frac{1}{2}\sum_{k=0}^{10}\frac{-1}{\sin\left(\frac{7\pi}{6}+k\pi\right)}\right]$

$= \sec^{-1}\left[\frac{1}{2}\sum_{k=0}^{10}\frac{-1}{\sin\left((k+1)\pi+\frac{\pi}{6}\right)}\right]$

If k is an even integer, then

$\sin((k+1)\pi+\frac{\pi}{6}) = -\sin\frac{\pi}{6} = \frac{-1}{2}$

If k is an odd integer, then $\sin((k+1)\pi+\frac{\pi}{6})$

$= \sin\frac{\pi}{6} = \frac{1}{2}$

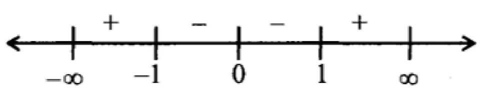
$\therefore \sum_{k=0}^{9}\sin\left((k+1)\pi+\frac{\pi}{6}\right) = 0$

Hence $\sec^{-1}\left[\frac{1}{2}\sum_{k=0}^{10}\frac{-1}{\sin\left((k+1)\pi+\frac{\pi}{6}\right)}\right]$

$= \sec^{-1}\left[\frac{1}{2}\left(\frac{-1}{-\frac{1}{2}}\right)\right] = \sec^{-1}(1) = 0$

45. 4.0

Explanation:



$3x^2+x-1=4\left|x^2-1\right|$

Case 1: If $x\in[-1,1]$

$3x^2+x-1=-4x^2+4$

$\Rightarrow 7x^2+x-5=0 \because D=141>0$

$\therefore$ Equation has two roots

Case 2: If $x\in(-\infty,-1]\cup[1,\infty)$

$3x^2+x-1=4x^2-4$

$\Rightarrow x^2-x-3=0 \because D=13>0$

$\therefore$ Equation has two roots So, total 4 roots

46. 18900.0

Explanation:

For A.P. $l_1, l_2, \ldots l_{100}$

Let T_1 = a and common difference = d_1

and similarly now for A.P. $w_1, w_2, \ldots w_{100}$

T_1 = b and common difference = d_2

$A_{51} - A_{50} = l_{51} w_{51} - l_{50} w_{50}$

$= (a + 50d_1)(b + 50d_2) - (a + 49d_1)(a + 49d_2)$

$= 50bd_1 + 50ad_2 + 2500d_1d_2 - 49ad_2 - 49bd_1 - 2401d_1d2$

$= bd_1 + ad_2 = 99d_1d_2 = 1000$

$= bd_1 + ad_2 = 1000 - 990 = 10$ ...(i) (As $d_1d_2 = 10$)

$\therefore A_{100} - A_{90} = l_{100}w_{100} - l_{90}w_{90}$

$= (a + 99d_1)(b + 99d_2) - (a + 89d_1)(b + 89d_2)$

$= 99bd_1 + 99ad_2 + 99^2d_1d_2 - 89bd_1 + 99ad_2 + 89^2d_1d_2$

$= 10(bd_1 + ad_2) + 1880d_1d_2$

$- 10(10) + 1880(10) = 1890$

47. 665

Explanation:

We make cases:

$X \rightarrow 2$ members

$Y \rightarrow 3$ members

$Z \rightarrow 4$ members

Case I: $S_1 \notin X, S_2 \in X$, (i.e., S_2 is in X, S_1 can be in Y or Z)

$1 \cdot {}^7C_1 \cdot \frac{\lfloor 7}{\lfloor 3 \lfloor 4} = 7 \cdot \frac{7 \cdot 6 \cdot 5}{6} = 245$

S_2 is there.

Case II: $S_1, S_2 \notin X, S_2 \notin Y$ (i.e. $S_2 \in Z, S_1$ can be in Y or Z)

${}^7C_2 \cdot \frac{\lfloor 6}{\lfloor 3 \lfloor 3} = 21 \cdot 20 = 420$

Hence, the total number of ways to make up team

$= 245 + 420 = 665$

48. 3

Explanation:

$\tan\theta = \cot 5\theta, \theta \neq \frac{n\pi}{5}$

$\Rightarrow \cos\theta \cos 5\theta - \sin 5\theta \sin\theta = 0 \Rightarrow \cos 6\theta = 0$

$\Rightarrow 6\theta = \frac{-5\pi}{2}, \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$\Rightarrow \theta = \frac{-5\pi}{12}, \frac{-\pi}{4}, \frac{-\pi}{12}, \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}$

Again $\sin 2\theta = \cos 4\theta = 1 - 2\sin^2 2\theta$

$\Rightarrow 2\sin^2 2\theta + \sin 2\theta - 1 = 0 \Rightarrow \sin 2\theta = -1, \frac{1}{2}$

$\Rightarrow 2\theta = \frac{-\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow \theta = \frac{-\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$

So, common solutions are $\theta = \frac{-\pi}{4} \cdot \frac{\pi}{12}$ and $\frac{5\pi}{12}$

$\therefore$ Number of solutions = 3